ENERGY TRANSMISSION THROUGH A MEDIUM WITH LOW OPTICAL DENSITY

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The transmission of radiant energy through a medium with low optical density is analyzed here. The radiative thermal conductivity of thin porous fiber layers was measured, and the results are shown.

The equation of radiant energy transmission through an absorbing plane layer bounded by gray diffusively emitting and reflecting surfaces, in the gray-body approximation and under conditions of local thermodynamic equilibrium, leads to the following expression:

$$q = \frac{\sigma T_1^4 - \sigma T_2^4}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1 + \frac{1}{2}\bar{\mu}\tau}. \quad (1)$$

A unity refractive index is assumed here and subsequently.

From a reconciliation between this and another well known expression for the radiative thermal conductivity

$$\lambda = \frac{16}{3}\sigma T^3 \bar{l} \quad (2)$$

it follows that $\bar{\mu} = 3/2$. Under the same conditions, (1) is identical to the result based on the Milne–Eddington approximation. That result can be obtained also directly by solving the equation of radiant energy transmission with the respective boundary conditions. The latter were first defined by G. L. Polyak [1] and then

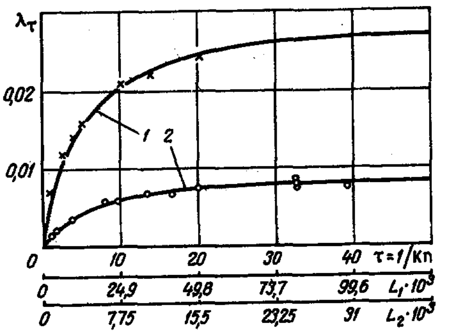

Fig. 1

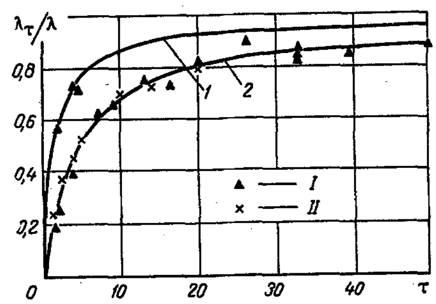

Fig. 2

Fig. 1. Effective radiative thermal conductivity λ_T (W/m · deg) as a function of the optical thickness τ (m) and of the geometrical thickness L (m), for a loose fibrous material (γ = 30 kg/m^3) with ε_e = 0.27: 1) Nylon fiber 30 μ in diameter; 2) superfine glass fiber 1-2 μ in diameter, calculated by formula (10) (solid lines).

Fig. 2. Ratio λ_T/λ as a function of the optical thickness, for a layer of density 30 kg/m^3, calculated by formula (10) (solid lines): 1) ε = 0.88; 2) ε = 0.27. Test points for superfine glass fiber (I) and Nylon fiber (II).

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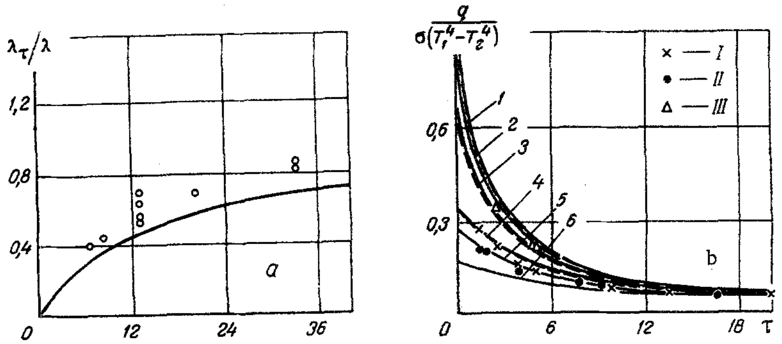

Fig. 3. Functions of the optical thickness of a layer: a) ratio λ_τ/λ for superfine glass fiber γ = 30 kg/m^3, polished aluminum as the cold surface with ε_e = 0.087, calculated for diffusive boundary surfaces (solid line) and with test points indicated. b) $q/\sigma(T_1^4-T_2^4)$, calculated from data in [3] (solid lines) and from data in [6] (dashed lines) with ε_e = 1 (1), 0.88 (2), 0.67 (3), 0.33 (4), 0.27 (5), 0.176 (6). Test points: Nylon fiber and ε = 0.28 (I), superfine glass fiber and ε_e = 0.26 (II), superfine glass fiber and ε_e = 0.88 (III). Curve 1 coincides with the data in [2, 7].

used on numerous occasions in several subsequent studies. It is only to be noted that the conventional stipulations concerning the quasiisotropy of incident radiation, usually found in the literature on this subject, are superfluous for making expression (2) and $\bar{\mu}$ = 3/2 in (1) valid. An analogous result will be obtained on the basis of the angular intensity distribution function, which can be expressed as a linear function of μ, while the intensity distribution of thermal fluxes of finite magnitude is not at all isotropic.

It has been reiterated [2-4] that the assumption of $\bar{\mu}$ in formulas like (1) being independent of the optical density of the layer is not strictly justified. Even the contrary has been asserted. In [5], for example, an attempt was made to prove that $\bar{\mu}$ is a function of τ under conventionally assumed restrictions. A comparison of radiant energy fluxes through thin optical layers calculated according to formulas like (1) on the basis of a constant $\bar{\mu}$ = 3/2 with results based on solutions to integral equation [6, 7] has shown an almost complete agreement over the entire range of τ values. This has been noted in [2, 4, et al.].

It should be noted that in all these studies the concept of radiant energy transmission was a simplified one. No consideration was given to the induced radiation of particles of the medium. The latter was assumed gray, without any scattering, and subject to conditions of local thermodynamic equilibrium.

In view of all this, much interest has been shown in direct experimental studies concerning the processes of radiant energy transmission through optically thin layers of real media. In this case it is impossible not to consider the effects having to do with emission and dispersion of radiant energy. It is reasonable that the first stage of such a study should cover low-intensity transmission processes, in which the conditions for local thermodynamic equilibrium are closely enough approximated. In the next stage, a study of conditions departing from equilibrium states will make it possible to determine also the applicability limits of the local-thermodynamic-equilibrium hypothesis.

Thus, when $T_1 \to T_2$, relation (1) can be rewritten as

$$q = \frac{4\,\sigma T^3 L}{\dfrac{1}{\varepsilon_e} + \dfrac{1}{2}\,\bar{\mu}\tau} \cdot \frac{dT}{dx} \tag{3}$$

or

$$\lambda_\tau = \frac{4\,\sigma T^3 L}{\dfrac{1}{\varepsilon_e} + \dfrac{1}{2}\,\bar{\mu}\tau}, \tag{4}$$

and, with the Knudsen radiation number, we have also

$$\lambda_\tau = \frac{\lambda}{1 + \dfrac{2\,\mathrm{Kn}}{\bar{\mu}\varepsilon_e}} \tag{5}$$

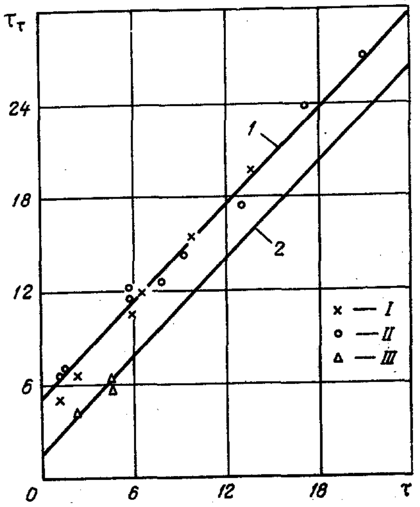

Fig. 4. Relation $\tau_T - \tau_T(\tau)$, calculated by Eq. (6) (solid lines) for $\varepsilon = 0.27$ (1), 0.88 (2). Test points: Nylon fiber and $\varepsilon_e = 0.28$ (I), superfine glass fiber and $\varepsilon_e = 0.28$ (II), superfine glass fiber and $\varepsilon_e = 0.88$ (III).

or, finally,

$$\tau_\tau = \frac{2}{\bar{\mu}\varepsilon_e} + \tau. \tag{6}$$

Expressions (3)-(6) are completely equivalent. The Knudsen number and the τ-parameter here are referred to continuum values of the mean-free-path of photons according to (2). The values of radiative thermal conductivity of layer here had been obtained experimentally under conditions of negligibly small boundary effects, i.e., at $L \gg \bar{l}$ and

$$\frac{d\lambda}{dL} = 0. \tag{7}$$

The authors have experimentally studied the thermal conductivity of loose fibrous masses with a varying optical thickness. The measurements were made under a high vacuum. In this way, the conducted heat could be reduced to a negligibly low level and conditions of radiative equilibrium could be ensured. We used superfine blow-grade glass fiber approximately 1-2 μ in diameter (average) and staple Nylon fiber approximately 30 μ in diameter. The optical density of a specimen was varied by varying the mass of material packed into the apparatus.

We applied the classical method of a plate and a steady thermal flux. The apparatus for measuring the effective thermal conductivity of specimens was made up of an electrical calorimeter separated from the housing by a layer of vacuum-shield grade thermal insulation (VSTI).

The presence of VSTI resulted in a appreciable reduction of stray thermal fluxes and ensured a satisfactory accuracy as well as an excellent repeatability of data. The temperatures were measured with Chromel–Copel thermocouples, a model R-306 low-resistance potentiometer, and a model M 195/1 galvanometer. The calorimeter heaters were energized from a stabilized power source. The drawn power was measured by the potentiometer method with these instruments. The "cold" operating surface of the apparatus was a shield cooled by running water. For materials of the boundary surface we used: oxidized aluminum, polished aluminum, oxidized copper, and varnish coatings on metallic layers.

The effective thermal conductivity was calculated by the formula

$$\lambda_\tau = \frac{QL}{F\Delta T}, \tag{8}$$

with the calorimeter heater power $Q = IV$, the temperature difference $\Delta T = T_1 - T_2$, and the surface area $F = 0.092\ \mathrm{m}^2$. The equivalent emissivity of the boundary surfaces was calculated by the formula for parallel infinitely large plates

$$\varepsilon_e = \frac{Q}{F\sigma(T_1^4 - T_2^4)}, \tag{9}$$

with the power Q measured without loose fiber in the apparatus. The power Q remained the same with the gap varying from 1 to 5 mm, an indication of negligible heat leakage through the ends. The error in determining the effective thermal conductivity did not exceed 5%.

With the aid of expression (4) and letting $\bar{\mu} = 3/2$, we have

$$\lambda_\tau = \frac{1}{\frac{1}{\lambda} + \frac{1}{4\varepsilon_e \sigma T^3 L}} \tag{10}$$

or

$$\lambda = \frac{1}{\frac{1}{\lambda_\tau} - \frac{1}{4\varepsilon_e \sigma T^3 L}}, \tag{11}$$

where $T = (T_1 + T_2)/2$.

The function $\lambda_{\tau} = \lambda_{\tau}(\tau)$ according to (10) is shown in Fig. 1 with $\varepsilon_e = 0.27$ as measured. Test values of effective thermal conductivity have also been plotted here. According to the data in Fig. 2, the differences between diffractivities of the test materials did not affect the trend of the curves for thin layers. The continuum characteristics of our test materials (Nylon fiber $\gamma = 30$ kg/m^3 and $\lambda = 0.031$ W/m·deg at T = 343°K, $\bar{l} = 2.52$ mm; superfine glass fiber $\gamma = 100$ kg/m^3 and $\lambda = 0.0028$ W/m·deg at T = 343°K, $\bar{l} = 0.23$ mm) determined from relation (11) and from test points for λ_{τ} agree with those obtained by direct measurements of the thermal conductivity of vacuumized optically dense layers.

The results indicate that relations (5), (10), and (11) do, within a sufficiently close approximation, describe the dependence of λ_{τ} on both the geometrical and the optical thickness of a layer. The differences between measured and calculated $\lambda_{\tau} = \lambda_{\tau}(\tau)$ values lie within the limits of test accuracy. The preceding discussion applies to transmission through layers with diffusively reflecting and emitting boundary surfaces. The use of polished aluminum as the boundary (cold) surface brought about a marked departure from relation (10) (Fig. 3a).

This result is quite a legitimate one. The emission of most energy at low angles to the surface, which is characteristic of polished metals, leads to an appreciable increase in the effective optical density of a thin layer. The data in Fig. 3b for diffusive surfaces have been compared with those obtained by R. F. Probstein [2], by M. A. Heaslet and F. B. Fuller [6], and by C. M. Usiskin and E. M. Sparrow [7].

Thus, the outcome of the experiments indicates that, when diffraction occurs at gray surfaces bounding a thin layer, the value $\bar{\mu} = 3/2$ may be assumed the same for every optical thickness, i.e., the boundary effects cause the optical density of a layer to increase by a constant amount independent of τ (Fig. 4).

NOTATION

q	is the thermal flux density, W/m^2;
σ	is the Stefan–Boltzmann constant;
ε_1, ε_2, ε_e	are the emissivity of surface 1 and surface 2 respectively, and equivalent emissivity of the system of both surfaces;
T_1, T_2	are the temperature of surface 1 and surface 2 respectively, °K;
$\underline{T} = (T_1 + T_2)/2$	is the average temperature of layer, °K;
$\bar{\mu}$	is the coefficient accounting for the angular distribution of incident radiation intensity;
τ	is the optical thickness (density) of layer;
τ_T	is the fictitious optical thickness of layer, according to (6);
λ, λ_{τ}	are the radiative thermal conductivity of an optically thick layer and of an optically thin layer respectively, W/m·deg;
$\bar{l}$	is the mean-free-path of photons in a medium, mm;
x	is the linear space coordinate, m;
L	is the geometrical thickness of layer, m;
Kn	is the Knudsen radiation number;
γ	is the density of specimen material, kg/m^3;
Q	is the power supplied by electrical heater, W;
F	is the area of active surface, m^2;
$\mu = \cos\theta$	is the angle with the normal to the surface.

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